

## Problem 12

In each of Problems 7 through 14, verify that each given function is a solution of the differential equation.

$$t^2 y'' + 5ty' + 4y = 0, \quad t > 0; \quad y_1(t) = t^{-2}, \quad y_2(t) = t^{-2} \ln t$$

### Solution

$$\begin{aligned} t^2 y_1'' + 5ty_1' + 4y_1 &\stackrel{?}{=} 0 \\ t^2 \frac{d^2}{dt^2}(t^{-2}) + 5t \frac{d}{dt}(t^{-2}) + 4(t^{-2}) &\stackrel{?}{=} 0 \\ t^2(-2)(-3)t^{-4} + 5t(-2)t^{-3} + 4t^{-2} &\stackrel{?}{=} 0 \\ 6t^{-2} - 10t^{-2} + 4t^{-2} &\stackrel{?}{=} 0 \\ 0 &= 0 \end{aligned}$$

The first solution is verified.

$$\begin{aligned} t^2 y_2'' + 5ty_2' + 4y_2 &\stackrel{?}{=} 0 \\ t^2 \frac{d^2}{dt^2}(t^{-2} \ln t) + 5t \frac{d}{dt}(t^{-2} \ln t) + 4(t^{-2} \ln t) &\stackrel{?}{=} 0 \\ t^2(6t^{-4} \ln t - 2t^{-4} - 3t^{-4}) + 5t(-2t^{-3} \ln t + t^{-3}) + 4t^{-2} \ln t &\stackrel{?}{=} 0 \\ 6t^{-2} \ln t - \cancel{5t^{-2}} - 10t^{-2} \ln t + \cancel{5t^{-2}} + 4t^{-2} \ln t &\stackrel{?}{=} 0 \\ 0 &= 0 \end{aligned}$$

The second solution is verified.