

Problem 14

In each of Problems 7 through 14, verify that each given function is a solution of the differential equation.

$$y' - 2ty = 1; \quad y = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$$

Solution

$$\begin{aligned}
 y' - 2ty &\stackrel{?}{=} 1 \\
 \frac{d}{dt} \left(e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2} \right) - 2t \left(e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2} \right) &\stackrel{?}{=} 1 \\
 \left(2te^{t^2} \int_0^t e^{-s^2} ds + e^{t^2} e^{-t^2} + 2te^{t^2} \right) - 2te^{t^2} \int_0^t e^{-s^2} ds - 2te^{t^2} &\stackrel{?}{=} 1 \\
 \cancel{2te^{t^2} \int_0^t e^{-s^2} ds} + 1 + \cancel{2te^{t^2}} - \cancel{2te^{t^2} \int_0^t e^{-s^2} ds} - \cancel{2te^{t^2}} &\stackrel{?}{=} 1 \\
 1 &= 1
 \end{aligned}$$

The solution is verified.