Problem 15

In each of Problems 15 through 18, determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

$$y' + 2y = 0$$

Solution

Because the terms on the left have constant coefficients, the solution to the ODE is of the form $y = e^{rt}$. Substitute it into the equation to determine r.

$$\begin{aligned} y' + 2y &= 0 \\ \frac{d}{dt}(e^{rt}) + 2(e^{rt}) &= 0 \\ re^{rt} + 2e^{rt} &= 0 \end{aligned}$$

Divide both sides by e^{rt}.

$$\begin{aligned} r + 2 &= 0 \\ r &= -2 \end{aligned}$$

Therefore, e^{-2t} is a solution to the ODE. The general solution is

$$y(t) = Ae^{-2t},$$

where A is an arbitrary constant.