Problem 13

In each of Problems 7 through 14, verify that each given function is a solution of the differential equation.

$$y'' + y = \sec t, \quad 0 < t < \pi/2; \qquad y = (\cos t)\ln\cos t + t\sin t$$

Solution

$$
\begin{aligned}
y'' + y &\stackrel{?}{=} \sec t \\
\frac{d^2}{dt^2}[(\cos t)\ln\cos t + t\sin t] + [(\cos t)\ln\cos t + t\sin t] &\stackrel{?}{=} \sec t \\
\frac{d}{dt}\left[(-\sin t)\ln\cos t + (\cos t)\frac{1}{\cos t}(-\sin t) + \sin t + t\cos t\right] + (\cos t)\ln\cos t + t\sin t &\stackrel{?}{=} \sec t \\
\frac{d}{dt}[(-\sin t)\ln\cos t + t\cos t] + (\cos t)\ln\cos t + t\sin t &\stackrel{?}{=} \sec t \\
\left[(-\cos t)\ln\cos t + (-\sin t)\frac{1}{\cos t}(-\sin t) + \cos t - t\sin t\right] + (\cos t)\ln\cos t + t\sin t &\stackrel{?}{=} \sec t \\
-\cancel{(\cos t)\ln\cos t} + \frac{\sin^2 t}{\cos t} + \cos t - \cancel{t\sin t} + \cancel{(\cos t)\ln\cos t} + \cancel{t\sin t} &\stackrel{?}{=} \sec t \\
\frac{\sin^2 t}{\cos t} + \cos t &\stackrel{?}{=} \sec t \\
\frac{\sin^2 t + \cos^2 t}{\cos t} &\stackrel{?}{=} \sec t \\
\frac{1}{\cos t} &\stackrel{?}{=} \sec t \\
\sec t &= \sec t
\end{aligned}
$$

The solution is verified.