

## Problem 16

In each of Problems 15 through 18, determine the values of  $r$  for which the given differential equation has solutions of the form  $y = e^{rt}$ .

$$y'' - y = 0$$

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### Solution

Because the terms on the left have constant coefficients, the solution to the ODE is of the form  $y = e^{rt}$ . Substitute it into the equation to determine  $r$ .

$$\begin{aligned}y'' - y &= 0 \\ \frac{d^2}{dt^2}(e^{rt}) - (e^{rt}) &= 0 \\ r^2 e^{rt} - e^{rt} &= 0\end{aligned}$$

Divide both sides by  $e^{rt}$ .

$$\begin{aligned}r^2 - 1 &= 0 \\ r = -1 \quad \text{or} \quad r &= 1\end{aligned}$$

Therefore,  $e^{-t}$  and  $e^t$  are two solutions to the ODE. The general solution is

$$y(t) = C_1 e^{-t} + C_2 e^t,$$

where  $C_1$  and  $C_2$  are arbitrary constants.