Problem 17

In each of Problems 15 through 18, determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

$$y'' + y' - 6y = 0$$

Solution

Because the terms on the left have constant coefficients, the solution to the ODE is of the form $y = e^{rt}$. Substitute it into the equation to determine r.

$$\begin{aligned} y'' + y' - 6y &= 0 \\ \frac{d^2}{dt^2}(e^{rt}) + \frac{d}{dt}(e^{rt}) - 6(e^{rt}) &= 0 \\ r^2 e^{rt} + re^{rt} - 6e^{rt} &= 0 \end{aligned}$$

Divide both sides by e^{rt}.

$$r^2 + r - 6 = 0$$

$$(r + 3)(r - 2) = 0$$

$$r = -3 \quad \text{or} \quad r = 2$$

Therefore, e^{-3t} and e^{2t} are two solutions to the ODE. The general solution is

$$y(t) = C_1 e^{-3t} + C_2 e^{2t},$$

where C_1 and C_2 are arbitrary constants.