Problem 18

In each of Problems 15 through 18, determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

$$y''' - 3y'' + 2y' = 0$$

Solution

Because the terms on the left have constant coefficients, the solution to the ODE is of the form $y = e^{rt}$. Substitute it into the equation to determine r.

$$\begin{aligned} y''' - 3y'' + 2y' &= 0 \\ \frac{d^3}{dt^3}(e^{rt}) - 3\frac{d^2}{dt^2}(e^{rt}) + 2\frac{d}{dt}(e^{rt}) &= 0 \\ r^3 e^{rt} - 3r^2 e^{rt} + 2re^{rt} &= 0 \end{aligned}$$

Divide both sides by e^{rt}.

$$r^3 - 3r^2 + 2r = 0$$

$$r(r-1)(r-2) = 0$$

$$r = 0 \quad \text{or} \quad r = 1 \quad \text{or} \quad r = 2$$

Therefore, 1 and e^t and e^{2t} are three solutions to the ODE. The general solution is

$$y(t) = C_1 + C_2 e^t + C_3 e^{2t},$$

where C_1 and C_2 and C_3 are arbitrary constants.