Problem 29

Follow the steps indicated here to derive the equation of motion of a pendulum, Eq. (12) in the text. Assume that the rod is rigid and weightless, that the mass is a point mass, and that there is no friction or drag anywhere in the system.

(a) Assume that the mass is in an arbitrary displaced position, indicated by the angle θ. Draw a free-body diagram showing the forces acting on the mass.

(b) Apply Newtons law of motion in the direction tangential to the circular arc on which the mass moves. Then the tensile force in the rod does not enter the equation. Observe that you need to find the component of the gravitational force in the tangential direction. Observe also that the linear acceleration, as opposed to the angular acceleration, is $Ld^2\theta/dt^2$, where L is the length of the rod.

(c) Simplify the result from part (b) to obtain Eq. (12) in the text.

Solution

Part (a)

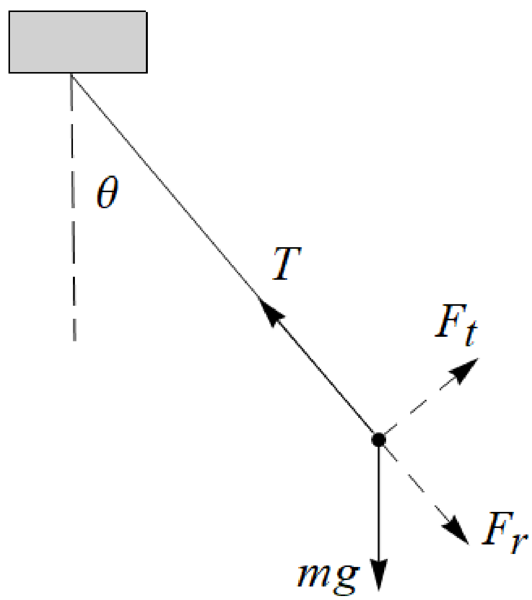

Figure 1: This figure shows the free-body diagram of the pendulum. The two forces to consider here are the tension in the rod and the gravitational force. F_r represents the net force in the radial direction, and F_t represents the net force in the tangential (positive-θ) direction.

Part (b)

Newton's second law states that the sum of the forces is equal to mass times acceleration.

$$\sum \mathbf{F} = m\mathbf{a}$$

If we use the radial and tangential directions, then this vector equation results in the following two scalar equations.

$$\sum F_r = ma_r = m\frac{v^2}{L} = \frac{m}{L}\left(L\frac{d\theta}{dt}\right)^2 = mL\left(\frac{d\theta}{dt}\right)^2$$
$$\sum F_t = ma_t = m\frac{dv}{dt} = m\frac{d}{dt}\left(L\frac{d\theta}{dt}\right) = mL\frac{d^2\theta}{dt^2}$$

Applying these two equations, we get

$$-T + mg\cos\theta = mL\left(\frac{d\theta}{dt}\right)^2 \tag{1}$$
$$-mg\sin\theta = mL\frac{d^2\theta}{dt^2}. \tag{2}$$

Part (c)

Divide both sides of equation (2) by mL

$$-\frac{g}{L}\sin\theta = \frac{d^2\theta}{dt^2}$$

and bring both terms to one side.

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \tag{12}$$

This is equation (12) in the textbook.