Problem 28

Consider the initial value problem

$$y' + \frac{2}{3}y = 1 - \frac{1}{2}t, \qquad y(0) = y_0.$$

Find the value of y_0 for which the solution touches, but does not cross, the t-axis.

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \left\langle 1, 1 - \frac{1}{2}t - \frac{2}{3}y \right\rangle dt$$

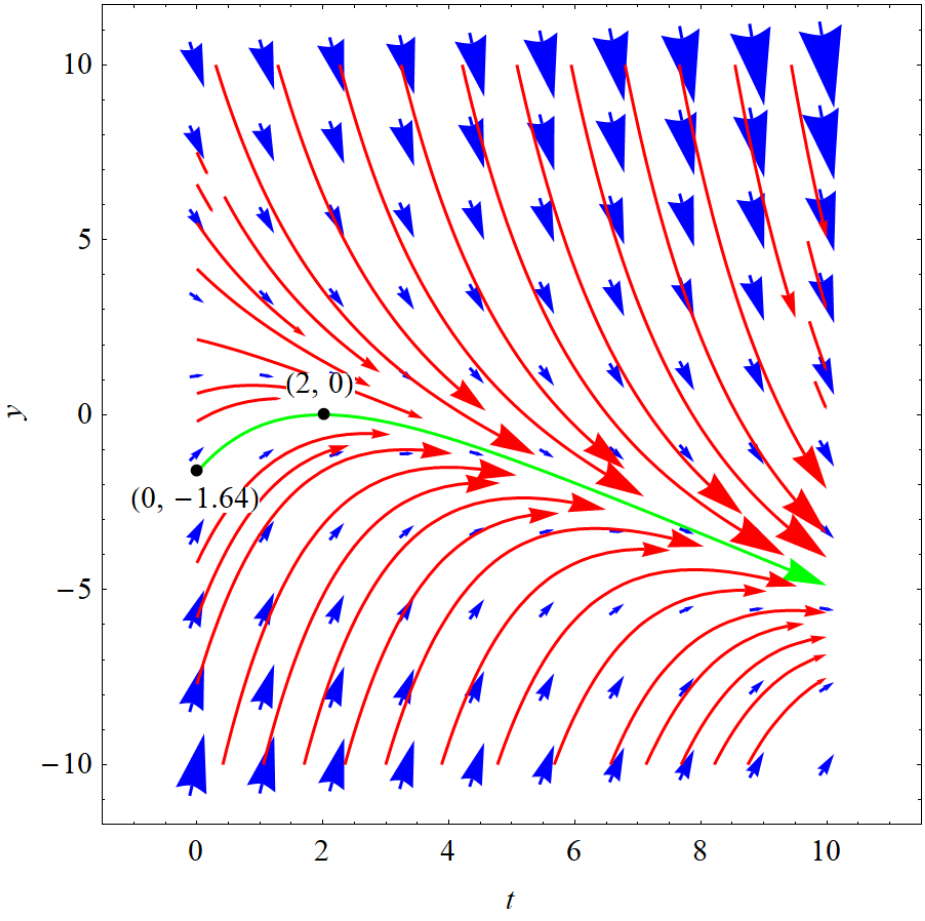

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. In particular, we are interested in the solution touching the line $y = 0$; it is labelled in green. The point at which it touches the line $t = 0$ is approximately $(0, -1.64)$.