Problem 13

In each of Problems 13 through 20, find the solution of the given initial value problem.

$$y' - y = 2te^t, \qquad y(0) = 1$$

Solution

This is a first-order linear inhomogeneous equation, so it can be solved with an integrating factor.

$$I(t) = \exp\left[\int^t (-1)\,ds\right] = e^{-t}$$

Multiply both sides of the ODE by $I(t)$.

$$e^{-t}y' - e^{-t}y = 2t$$

The left side can be written as $d/dt(Iy)$ by the product rule.

$$\frac{d}{dt}(e^{-t}y) = 2t$$

Integrate both sides with respect to t.

$$e^{-t}y = t^2 + C$$

Multiply both sides by e^t to solve for $y(t)$.

$$y(t) = e^t(t^2 + C)$$

Apply the initial condition $y(0) = 1$ here to determine C.

$$y(0) = 1(0 + C) = 1 \quad \rightarrow \quad C = 1$$

Therefore,

$$y(t) = e^t(t^2 + 1).$$