Problem 2

In each of Problems 1 through 8, solve the given differential equation.

$$y' = x^2/y(1+x^3)$$

Solution

This ODE is separable because it is of the form $y' = f(x)g(y)$, so it can be solved by separating variables.

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

Bring the terms with y to the left and bring the terms with x to the right.

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

Integrate both sides.

$$\int y\,dy = \int \frac{x^2}{1+x^3}\,dx \tag{1}$$

Use a substitution to evaluate the integral on the right.

$$u = 1 + x^3$$
$$du = 3x^2\,dx \quad \rightarrow \quad \frac{du}{3} = x^2\,dx$$

Equation (1) becomes

$$\begin{aligned} \frac{y^2}{2} &= \int \frac{1}{u}\frac{du}{3} \\ &= \frac{1}{3}\ln|u| + C \\ &= \frac{1}{3}\ln|1+x^3| + C. \end{aligned}$$

Now solve for y.

$$y^2 = \frac{2}{3}\ln|1+x^3| + 2C$$
$$y(x) = \pm\sqrt{\frac{2}{3}\ln|1+x^3| + 2C}$$

Therefore, using a new constant C_1 for $2C$,

$$y(x) = \pm\sqrt{\frac{2}{3}\ln|1+x^3| + C_1}.$$