

Problem 4

In each of Problems 1 through 8, solve the given differential equation.

$$y' = (3x^2 - 1)/(3 + 2y)$$

Solution

This ODE is separable because it is of the form $y' = f(x)g(y)$, so it can be solved by separating variables.

$$\frac{dy}{dx} = \frac{3x^2 - 1}{3 + 2y}$$

Bring the terms with y to the left and bring the terms with x to the right.

$$(3 + 2y) dy = (3x^2 - 1) dx$$

Integrate both sides.

$$\int (3 + 2y) dy = \int (3x^2 - 1) dx$$
$$3y + y^2 = x^3 - x + C$$

Now solve the equation for y .

$$y^2 + 3y - (x^3 - x + C) = 0$$
$$y(x) = \frac{-3 \pm \sqrt{9 + 4(x^3 - x + C)}}{2}.$$