Problem 19

In each of Problems 9 through 20:

(a) Find the solution of the given initial value problem in explicit form.

(b) Plot the graph of the solution.

(c) Determine (at least approximately) the interval in which the solution is defined.

$$\sin 2x\,dx + \cos 3y\,dy = 0, \qquad y(\pi/2) = \pi/3$$

Solution

Bring $\sin 2x\,dx$ to the right side.

$$\cos 3y\,dy = -\sin 2x\,dx$$

The variables, x and y, are separated already. Integrate both sides.

$$\int \cos 3y\,dy = -\int \sin 2x\,dx$$

$$\frac{1}{3}\sin 3y = \frac{1}{2}\cos 2x + C$$

Apply the initial condition to determine C.

$$\frac{1}{3}\sin\left[3\left(\frac{\pi}{3}\right)\right] = \frac{1}{2}\cos\left[2\left(\frac{\pi}{2}\right)\right] + C \quad \rightarrow \quad 0 = -\frac{1}{2} + C \quad \rightarrow \quad C = \frac{1}{2}$$

As a result,

$$\frac{1}{3}\sin 3y = \frac{1}{2}\cos 2x + \frac{1}{2}.$$

Multiply both sides by 3.

$$\sin 3y = \frac{3}{2}\cos 2x + \frac{3}{2}$$

Use the identity $\sin z = -\sin(z - \pi)$ so that the initial condition remains satisfied.

$$-\sin(3y - \pi) = \frac{3}{2}\cos 2x + \frac{3}{2}$$

$$\sin(3y - \pi) = -\frac{3}{2}(\cos 2x + 1)$$

Take the inverse sine of both sides.

$$3y - \pi = \sin^{-1}\left[-\frac{3}{2}(\cos 2x + 1)\right]$$

Use the identity $\cos 2x + 1 = 2\cos^2 x$.

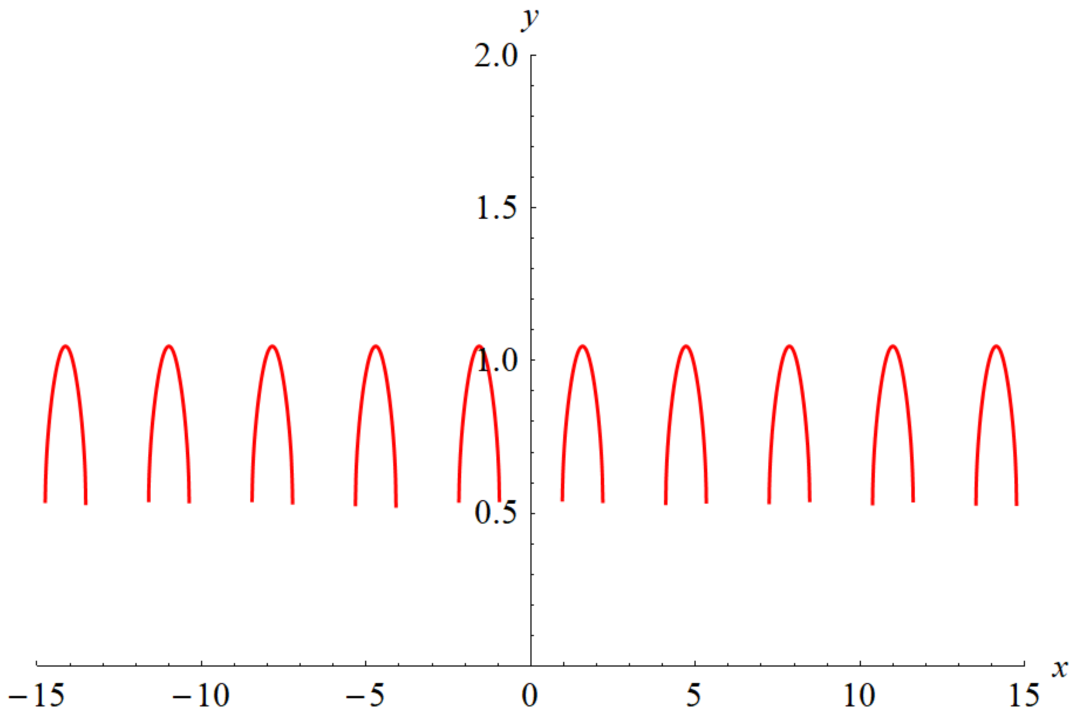
$$3y = \pi - \sin^{-1}\left[\frac{3}{2}(2\cos^2 x)\right]$$

Therefore,

$$y(x) = \frac{1}{3}\left[\pi - \sin^{-1}(3\cos^2 x)\right].$$

Part (b)

Below is a plot of $y(x)$ versus x.



Part (c)

The solution we found is only valid along the curve that passes through $x = \frac{\pi}{2}$ because $x = \frac{\pi}{2}$ is where the initial condition is given.

$$0 < 3\cos^2 x < 1$$

$$0 < \cos^2 x < \frac{1}{3}$$

$$-\frac{1}{\sqrt{3}} < \cos x < \frac{1}{\sqrt{3}}$$

$$0.96 \lesssim x \lesssim 2.19$$