# Problem 20

In each of Problems 9 through 20:

(a) Find the solution of the given initial value problem in explicit form.

(b) Plot the graph of the solution.

(c) Determine (at least approximately) the interval in which the solution is defined.

$$y^2(1-x^2)^{1/2}\,dy = \arcsin x\,dx, \qquad y(0)=1$$

---

**Solution**

## Part (a)

Divide both sides by $(1-x^2)^{1/2}$ to separate variables.

$$y^2\,dy = \frac{\arcsin x}{(1-x^2)^{1/2}}\,dx$$

Integrate both sides.

$$\int y^2\,dy = \int \frac{\arcsin x}{(1-x^2)^{1/2}}\,dx$$

Let $u = \arcsin x$ in the integral on the right. Then $du = dx/(1-x^2)^{1/2}$.

$$\frac{y^3}{3} = \int^{\arcsin x} u\,du + C$$

$$\frac{y^3}{3} = \frac{1}{2}(\arcsin x)^2 + C$$

Use the initial condition here to determine $C$.

$$\frac{1^3}{3} = \frac{1}{2}(\arcsin 0)^2 + C \quad \rightarrow \quad C = \frac{1}{3}$$

As a result,

$$\frac{y^3}{3} = \frac{1}{2}(\arcsin x)^2 + \frac{1}{3}.$$

Multiply both sides by 3.

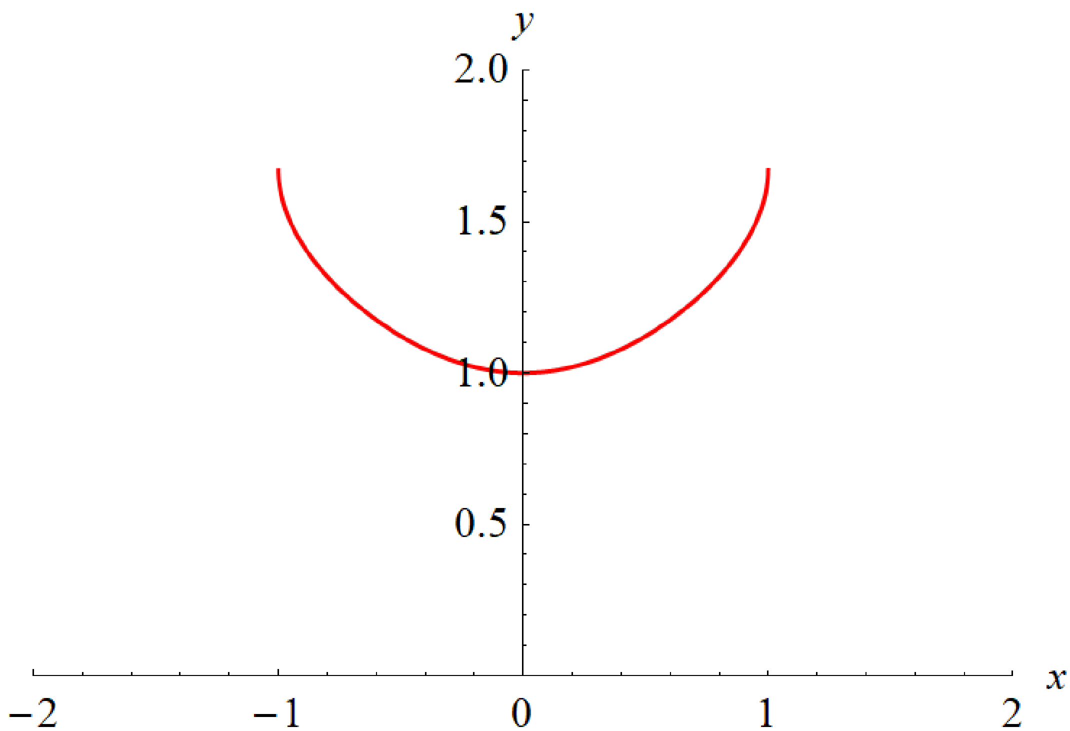
$$y^3 = \frac{3}{2}(\arcsin x)^2 + 1$$

Therefore,

$$y(x) = \sqrt[3]{\frac{3}{2}(\arcsin x)^2 + 1}.$$

## Part (b)

Below is a plot of $y(x)$ versus $x$.




## Part (c)

The solution we found is only valid along the curve that passes through $x = 0$ (the $y$-axis) because $x = 0$ is where the initial condition is given. The domain of $\arcsin x$ is $-1 < x < 1$.