

## Problem 7

In each of Problems 1 through 8, solve the given differential equation.

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$$

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### Solution

This ODE is separable because it is of the form  $y' = f(x)g(y)$ , so it can be solved by separating variables. Bring the terms with  $y$  to the left and bring the terms with  $x$  to the right.

$$(y + e^y) dy = (x - e^{-x}) dx$$

Integrate both sides.

$$\begin{aligned}\int (y + e^y) dy &= \int (x - e^{-x}) dx \\ \int y dy + \int e^y dy &= \int x dx - \int e^{-x} dx \\ \frac{y^2}{2} + e^y &= \frac{x^2}{2} + e^{-x} + C\end{aligned}$$

Since  $y$  cannot be solved for explicitly, we leave the equation as it is. The solution is implicit.