Problem 20

A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m high. Neglect air resistance.

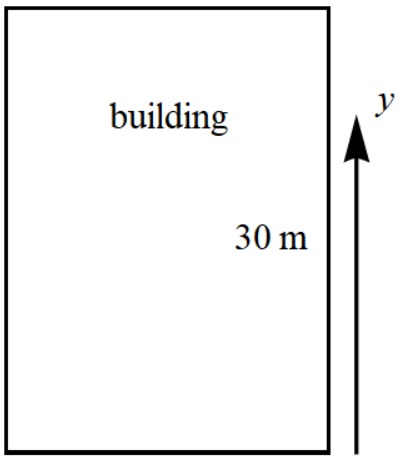
(a) Find the maximum height above the ground that the ball reaches.

(b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

(c) Plot the graphs of velocity and position versus time.

Solution

Use the following coordinate system for this problem.



Part (a)

Use the kinematic formula,

$$v^2 = v_0^2 + 2a\Delta y.$$

At the ball's maximum height, $v = 0$, and since gravity is pointing down, $a = -g$. Solve the resulting equation for Δy.

$$0 = v_0^2 - 2g\Delta y$$

$$\Delta y = \frac{v_0^2}{2g} \approx \frac{20^2}{2(9.81)} \approx 20.4 \text{ meters}$$

Δy is how high above the roof the ball goes. Since we want to know how high above the ground it is, we have to add the building's height to it.

$$h \approx 30 + 20.4 = 50.4 \text{ meters}$$

Part (b)

Use the kinematic formula,

$$y = y_0 + v_0 t + \frac{1}{2}at^2,$$

taking the initial height to be $y_0 = 30$, the initial velocity to be $v_0 = 20$, and the acceleration to be $a = -g$.

$$y = 30 + 20t - \frac{1}{2}gt^2$$

Set $y = 0$ and solve the resulting equation for t to find when the ball hits the ground.

$$0 = 30 + 20t - \frac{g}{2}t^2$$

Use the quadratic formula.

$$t = \frac{-20 \pm \sqrt{20^2 + 4\left(\frac{g}{2}\right)30}}{-g}$$

We choose the plus sign because t must be a positive number.

$$t \approx 5.24 \text{ seconds}$$

Part (c)

The position of the ball as a function of time is

$$y(t) = 30 + 20t - \frac{1}{2}gt^2,$$

and the velocity of the ball is $v_y = dy/dt$, or $v_y = v_0 + at$.

$$v_y(t) = 20 - gt$$