

Problem 14

Suppose that a certain population has a growth rate that varies with time and that this population satisfies the differential equation

$$dy/dt = (0.5 + \sin t)y/5.$$

- (a) If $y(0) = 1$, find (or estimate) the time τ at which the population has doubled. Choose other initial conditions and determine whether the doubling time τ depends on the initial population.
- (b) Suppose that the growth rate is replaced by its average value $1/10$. Determine the doubling time τ in this case.
- (c) Suppose that the term $\sin t$ in the differential equation is replaced by $\sin 2\pi t$; that is, the variation in the growth rate has a substantially higher frequency. What effect does this have on the doubling time τ ?
- (d) Plot the solutions obtained in parts (a), (b), and (c) on a single set of axes.