Problem 3

In each of Problems 1 through 6, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

$$y' + (\tan t)y = \sin t, \qquad y(\pi) = 0$$

Solution

According to Theorem 2.4.1, a unique solution to

$$y' + p(t)y = g(t), \qquad y(t_0) = y_0$$

exists throughout any interval in t containing the point t_0 where the functions, $p(t)$ and $g(t)$, are continuous. $p(t)$ is discontinuous where cosine is continuous:

$$t = \frac{1}{2}(2n+1)\pi, \quad n = 0, \pm 1, \pm 2, \ldots.$$

Since $t_0 = \pi$ is between $\pi/2$ ($n = 0$) and $3\pi/2$ ($n = 1$), a unique solution will exist for $\pi/2 < t < 3\pi/2$.