Problem 11

In each of Problems 7 through 12, state where in the ty-plane the hypotheses of Theorem 2.4.2 are satisfied.

$$\frac{dy}{dt} = \frac{1+t^2}{3y-y^2}$$

Solution

According to Theorem 2.4.2, a unique solution to

$$y' = f(t,y), \qquad y(t_0) = y_0$$

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that f and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains (t_0, y_0). In this exercise

$$f(t,y) = \frac{1+t^2}{3y-y^2} = \frac{1+t^2}{y(3-y)} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{(0)(3y-y^2)-(1+t^2)(3-2y)}{(3y-y^2)^2} = -\frac{(1+t^2)(3-2y)}{y^2(3-y)^2}.$$

Both f and $\partial f/\partial y$ are discontinuous at $y = 0$ and $y = 3$. Therefore, the hypotheses of Theorem 2.4.2 are satisfied if $y \neq 0$ and $y \neq 3$.