Problem 23

(a) Show that $\phi(t) = e^{2t}$ is a solution of $y' - 2y = 0$ and that $y = c\phi(t)$ is also a solution of this equation for any value of the constant c.

(b) Show that $\phi(t) = 1/t$ is a solution of $y' + y^2 = 0$ for $t > 0$ but that $y = c\phi(t)$ is not a solution of this equation unless $c = 0$ or $c = 1$. Note that the equation of part (b) is nonlinear, while that of part (a) is linear.

Solution

Part (a)

Substitute $y = e^{2t}$ into the ODE to show that it is a solution.

$$(e^{2t})' - 2(e^{2t}) \stackrel{?}{=} 0$$

$$(2e^{2t}) - 2(e^{2t}) \stackrel{?}{=} 0$$

$$0 = 0$$

As a result, $y = e^{2t}$ satisfies

$$(e^{2t})' - 2(e^{2t}) = 0.$$

Multiply both sides by c.

$$c(e^{2t})' - 2c(e^{2t}) = 0$$

Because c is just a constant, it can be brought inside each set of parentheses.

$$(ce^{2t})' - 2(ce^{2t}) = 0$$

Therefore, $y = ce^{2t}$ is also a solution of the ODE.

Part (b)

Substitute $y = 1/t$ into the ODE to show that it is a solution.

$$\left(\frac{1}{t}\right)' + \left(\frac{1}{t}\right)^2 \stackrel{?}{=} 0$$

$$-\frac{1}{t^2} + \frac{1}{t^2} \stackrel{?}{=} 0$$

$$0 = 0$$

For $t > 0$, then, $y = 1/t$ is a solution, and it satisfies

$$\left(\frac{1}{t}\right)' + \left(\frac{1}{t}\right)^2 = 0.$$

Multiply both sides by c.

$$c\left(\frac{1}{t}\right)' + c\left(\frac{1}{t}\right)^2 = 0.$$

Because c is just a constant, it can be brought inside each set of parentheses.

$$\left(c\frac{1}{t}\right)' + \left(\sqrt{c}\frac{1}{t}\right)^2 = 0.$$

Therefore, $y = c(1/t)$ is not a solution of this equation unless $c = 0$ or $c = 1$. The reason is because the ODE is nonlinear.