

## Problem 10

In each of Problems 7 through 12, state where in the  $ty$ -plane the hypotheses of Theorem 2.4.2 are satisfied.

$$y' = (t^2 + y^2)^{3/2}$$

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### Solution

According to Theorem 2.4.2, a unique solution to

$$y' = f(t, y), \quad y(t_0) = y_0$$

exists in some interval  $t_0 - h < t < t_0 + h$  within  $\alpha < t < \beta$ , provided that  $f$  and  $\partial f/\partial y$  are continuous in a rectangle  $\alpha < t < \beta$ ,  $\gamma < y < \delta$  that contains  $(t_0, y_0)$ . In this exercise

$$f(t, y) = (t^2 + y^2)^{3/2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{3}{2}(t^2 + y^2)^{1/2}(2y) = 3y(t^2 + y^2)^{1/2}.$$

$f$  is continuous everywhere, and  $\partial f/\partial y$  is continuous everywhere. Therefore, the hypotheses of Theorem 2.4.2 are satisfied everywhere.