

Problem 24

Show that if $y = \phi(t)$ is a solution of $y' + p(t)y = 0$, then $y = c\phi(t)$ is also a solution for any value of the constant c .

Solution

Suppose that $y = \phi(t)$ is a solution of $y' + p(t)y = 0$. Then

$$[\phi(t)]' + p(t)[\phi(t)] = 0.$$

Multiply both sides by c , a constant.

$$c[\phi(t)]' + cp(t)[\phi(t)] = 0.$$

Bring the constant inside each set of square brackets.

$$[c\phi(t)]' + p(t)[c\phi(t)] = 0.$$

Therefore, $c\phi(t)$ satisfies the ODE and is a solution as well. The reason is because $y' + p(t)y = 0$ is a linear ODE.