Problem 4

In each of Problems 1 through 6, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

$$(4 - t^2)y' + 2ty = 3t^2, \qquad y(-3) = 1$$

Solution

According to Theorem 2.4.1, a unique solution to

$$y' + p(t)y = g(t), \qquad y(t_0) = y_0$$

exists throughout any interval in t containing the point t_0 where the functions, $p(t)$ and $g(t)$, are continuous. Divide both sides of the ODE by $4 - t^2$ to put it in standard form.

$$y' + \frac{2t}{4 - t^2}y = \frac{3t^2}{4 - t^2}$$

$p(t)$ and $g(t)$ are discontinuous at $t = \pm 2$. Since $t_0 = -3$ is to the left of -2 on the number line, a unique solution will exist for $-\infty < t < -2$.