Problem 6

In each of Problems 1 through 6, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

$$(\ln t)y' + y = \cot t, \qquad y(2) = 3$$

Solution

According to Theorem 2.4.1, a unique solution to

$$y' + p(t)y = g(t), \qquad y(t_0) = y_0$$

exists throughout any interval in t containing the point t_0 where the functions, $p(t)$ and $g(t)$, are continuous. Divide both sides of the ODE by $\ln t$ to put it in standard form.

$$y' + \frac{1}{\ln t}y = \frac{\cot t}{\ln t}$$

The logarithm is defined for $0 < t < \infty$. In $p(t)$ there is a discontinuity at $t = 1$, and in $g(t)$ there are discontinuities at $t = n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$. Since $t_0 = 2$ is between 1 and π ($n = 1$), a unique solution will exist for $1 < t < \pi$.