

Problem 8

In each of Problems 7 through 12, state where in the ty -plane the hypotheses of Theorem 2.4.2 are satisfied.

$$y' = (1 - t^2 - y^2)^{1/2}$$

Solution

According to Theorem 2.4.2, a unique solution to

$$y' = f(t, y), \quad y(t_0) = y_0$$

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that f and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains (t_0, y_0) . In this exercise

$$f(t, y) = (1 - t^2 - y^2)^{1/2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{1}{2}(1 - t^2 - y^2)^{-1/2}(-2y) = -\frac{y}{\sqrt{1 - t^2 - y^2}}.$$

f is continuous as long as $1 - t^2 - y^2 \geq 0$, and $\partial f/\partial y$ is continuous as long as $1 - t^2 - y^2 > 0$. Therefore, the hypotheses of Theorem 2.4.2 are satisfied if $1 - t^2 - y^2 > 0$, or $t^2 + y^2 < 1$.