Problem 9

In each of Problems 7 through 12, state where in the ty-plane the hypotheses of Theorem 2.4.2 are satisfied.

$$y' = \frac{\ln|ty|}{1 - t^2 + y^2}$$

Solution

According to Theorem 2.4.2, a unique solution to

$$y' = f(t, y), \qquad y(t_0) = y_0$$

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that f and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains (t_0, y_0). In this exercise

$$f(t, y) = \frac{\ln ty}{1 - t^2 + y^2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{\frac{1}{ty} \cdot t(1 - t^2 + y^2) - (2y)\ln ty}{(1 - t^2 + y^2)^2} = \frac{(1 - t^2 + y^2) - 2y^2 \ln ty}{y(1 - t^2 + y^2)^2}$$

in the first and third quadrants of the ty-plane and

$$f(t, y) = \frac{\ln(-ty)}{1 - t^2 + y^2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{\frac{1}{-ty} \cdot (-t)(1 - t^2 + y^2) - (2y)\ln(-ty)}{(1 - t^2 + y^2)^2} = \frac{(1 - t^2 + y^2) - 2y^2 \ln(-ty)}{y(1 - t^2 + y^2)^2}$$

in the second and fourth quadrants of the ty-plane. f is continuous as long as $|ty| > 0$ and $1 - t^2 + y^2 \neq 0$, and $\partial f/\partial y$ is continuous as long as $|ty| > 0$ and $1 - t^2 + y^2 \neq 0$. Therefore, the hypotheses of Theorem 2.4.2 are satisfied if $t \neq 0$ and $y \neq 0$ and $(1 - t^2 + y^2 < 0$ or $1 - t^2 + y^2 > 0)$.