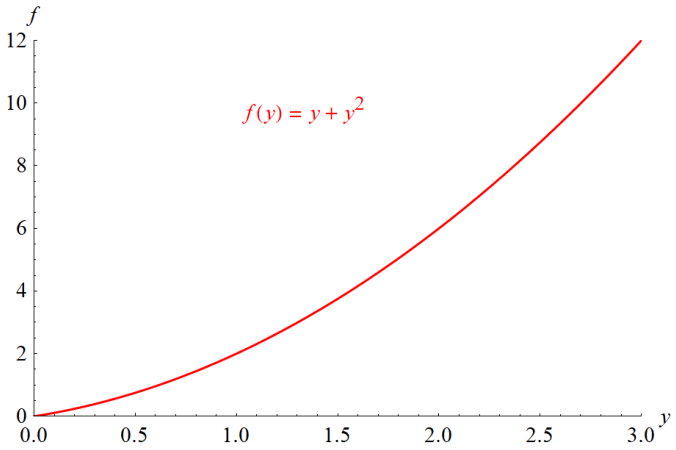
Problem 1

Problems 1 through 6 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions in the ty-plane.

$$dy/dt = ay + by^2, \qquad a > 0, \quad b > 0, \quad y_0 \geq 0$$

Solution

In this problem $f(y) = ay + by^2$. For $a = 1$ and $b = 1$, the graph of $f(y)$ versus y is shown below.



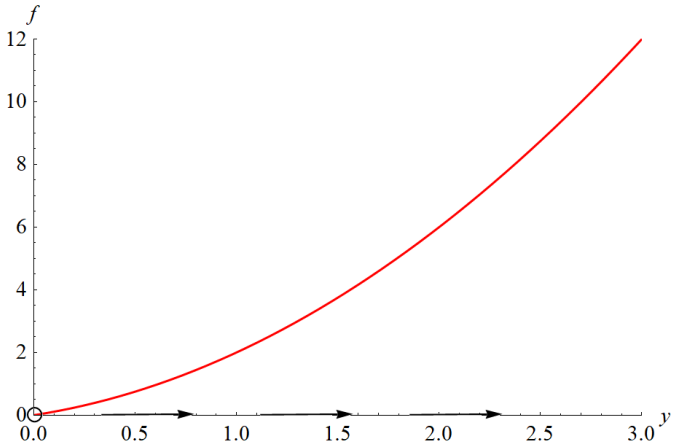
The equilibrium points are found by solving $f(y) = 0$ for y.

$$ay + by^2 = 0$$

$$y(a + by) = 0$$

$$y = \left\{-\frac{a}{b}, 0\right\}$$

Since $y_0 \geq 0$, only $y = 0$ is a relevant equilibrium point here. $f(y)$ is positive to the right of the equilibrium point, so $y = 0$ is unstable. It is represented by an open circle.



The arrows on the y-axis (phase line) indicate that y is increasing in time.

Some possible solution curves in the ty-plane for $t \geq 0$ and $y \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, y + y^2 \rangle$.