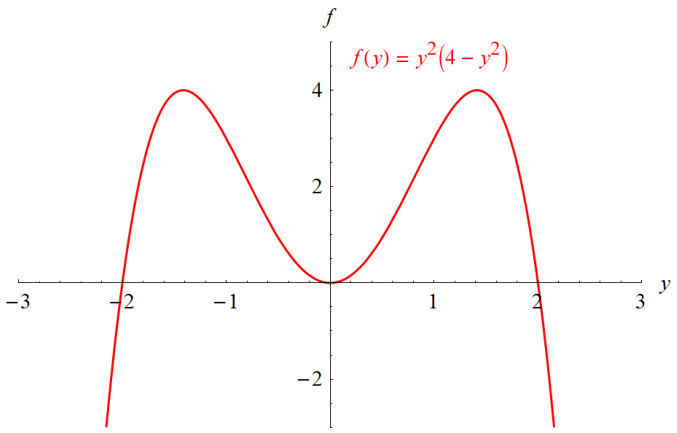
Problem 12

Problems 8 through 13 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points, and classify each one asymptotically stable, unstable, or semistable (see Problem 7). Draw the phase line, and sketch several graphs of solutions in the ty-plane.

$$dy/dt = y^2(4 - y^2), \qquad -\infty < y_0 < \infty$$

Solution

In this problem $f(y) = y^2(4 - y^2)$. Below is a graph of $f(y)$ versus y.



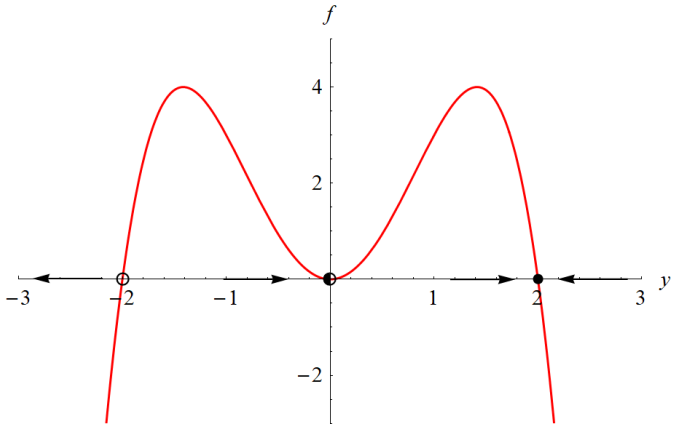
The equilibrium solutions are found by solving $f(y) = 0$ for y.

$$y^2(4 - y^2) = 0$$

$$y^2 = 0 \quad \text{or} \quad 4 - y^2 = 0$$

$$y = \{-2, 0, 2\}$$

As indicated below by the half-filled circle at $y = 0$, the equilibrium is stable from the left but unstable from the right. It is said to be semistable. $y = -2$ is an unstable equilibrium solution, and $y = 2$ is a stable equilibrium solution.



The arrows pointing left and right on the y-axis (phase line) mean that y is decreasing and increasing in time, respectively.

Some possible solution curves in the ty-plane for $t \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, y^2(4-y^2)\rangle$.