

Problem 16

Another equation that has been used to model population growth is the Gompertz¹⁴ equation

$$dy/dt = ry \ln(K/y),$$

where r and K are positive constants.

- (a) Sketch the graph of $f(y)$ versus y , find the critical points, and determine whether each is asymptotically stable or unstable.
- (b) For $0 \leq y \leq K$, determine where the graph of y versus t is concave up and where it is concave down.
- (c) For each y in $0 < y \leq K$, show that dy/dt as given by the Gompertz equation is never less than dy/dt as given by the logistic equation.

¹⁴Benjamin Gompertz (1779–1865) was an English actuary. He developed his model for population growth, published in 1825, in the course of constructing mortality tables for his insurance company.