

Problem 19

Consider a cylindrical water tank of constant cross section A . Water is pumped into the tank at a constant rate k and leaks out through a small hole of area a in the bottom of the tank. From Torricelli's principle in hydrodynamics (see Problem 6 in Section 2.3) it follows that the rate at which water flows through the hole is $\alpha a\sqrt{2gh}$, where h is the current depth of water in the tank, g is the acceleration due to gravity, and α is a contraction coefficient that satisfies $0.5 \leq \alpha \leq 1.0$.

- (a) Show that the depth of water in the tank at any time satisfies the equation

$$dh/dt = (k - \alpha a\sqrt{2gh})/A.$$

- (b) Determine the equilibrium depth h_e of water, and show that it is asymptotically stable. Observe that h_e does not depend on A .