

Problem 28

Chemical Reactions. A second order chemical reaction involves the interaction (collision) of one molecule of a substance P with one molecule of a substance Q to produce one molecule of a new substance X ; this is denoted by $P + Q \rightarrow X$. Suppose that p and q , where $p \neq q$, are the initial concentrations of P and Q , respectively, and let $x(t)$ be the concentration of X at time t . Then $p - x(t)$ and $q - x(t)$ are the concentrations of P and Q at time t , and the rate at which the reaction occurs is given by the equation

$$dx/dt = \alpha(p - x)(q - x), \quad (\text{i})$$

where α is a positive constant.

- (a) If $x(0) = 0$, determine the limiting value of $x(t)$ as $t \rightarrow \infty$ without solving the differential equation. Then solve the initial value problem and find $x(t)$ for any t .
- (b) If the substances P and Q are the same, then $p = q$ and Eq. (i) is replaced by

$$dx/dt = \alpha(p - x)^2. \quad (\text{ii})$$

If $x(0) = 0$, determine the limiting value of $x(t)$ as $t \rightarrow \infty$ without solving the differential equation. Then solve the initial value problem and determine $x(t)$ for any t .