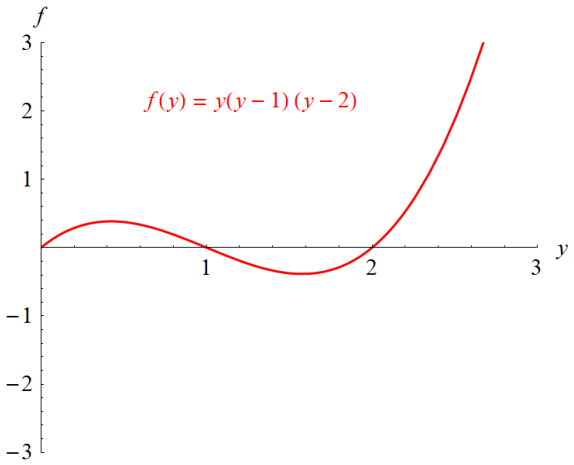
Problem 3

Problems 1 through 6 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions in the ty-plane.

$$dy/dt = y(y-1)(y-2), \qquad y_0 \geq 0$$

Solution

In this problem $f(y) = y(y-1)(y-2)$. Below is a graph of $f(y)$ versus y.

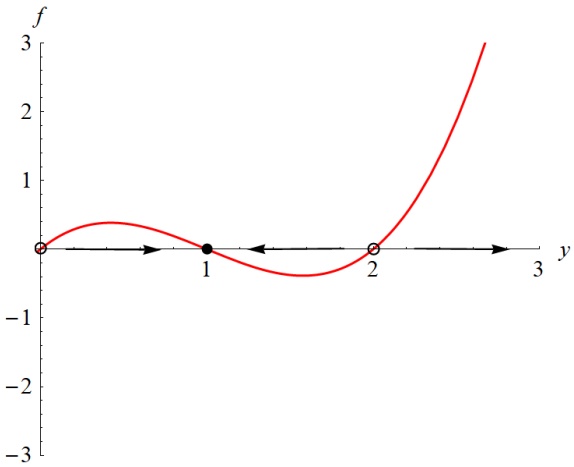


The equilibrium points are found by solving $f(y) = 0$ for y.

$$y(y-1)(y-2) = 0$$

$$y = \{0, 1, 2\}$$

The open and closed circles represent unstable and stable equilibrium points, respectively. $y = 0$ and $y = 2$ are unstable while $y = 1$ is stable.



The arrows pointing left and right on the y-axis (phase line) mean that y is decreasing and increasing in time, respectively.

Some possible solution curves in the ty-plane for $t \geq 0$ and $y \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, y(y-1)(y-2) \rangle$.