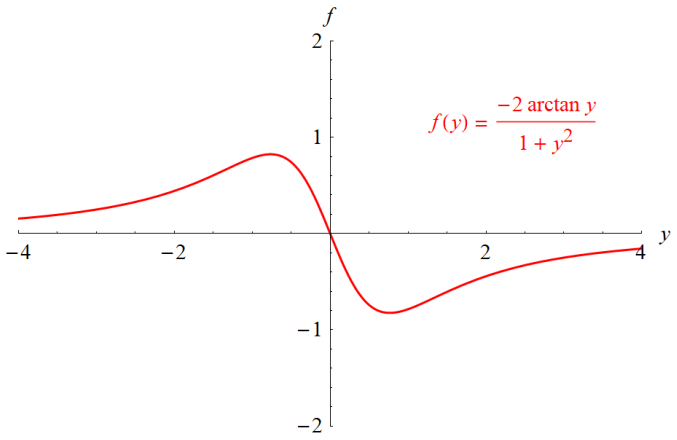
Problem 6

Problems 1 through 6 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions in the ty-plane.

$$dy/dt = -2(\arctan y)/(1+y^2), \qquad -\infty < y_0 < \infty$$

Solution

In this problem $f(y) = -2(\arctan y)/(1+y^2)$. Below is a graph of $f(y)$ versus y.

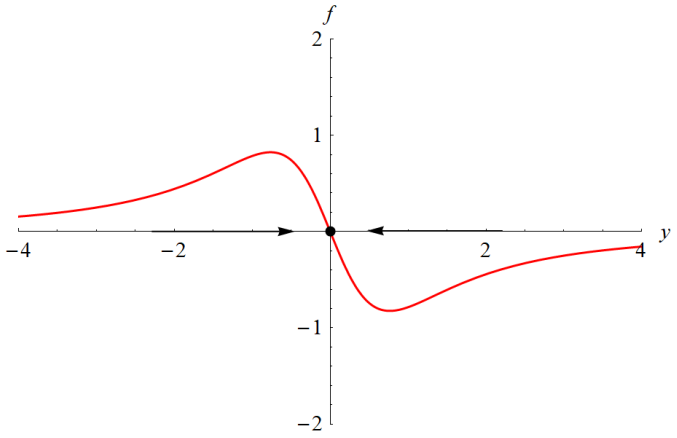


The equilibrium points are found by solving $f(y) = 0$ for y.

$$-2(\arctan y)/(1+y^2) = 0$$

$$y = \{0\}$$

As indicated below by the closed circle, $y = 0$ is stable.



The arrows pointing left and right on the y-axis (phase line) mean that y is decreasing and increasing in time, respectively.

Some possible solution curves in the ty-plane for $t \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, -2(\arctan y)/(1+y^2) \rangle$.