Problem 7

Semistable Equilibrium Solutions. Sometimes a constant equilibrium solution has the property that solutions lying on one side of the equilibrium solution tend to approach it, whereas solutions lying on the other side depart from it (see Figure 2.5.9). In this case the equilibrium solution is said to be **semistable**.

(a) Consider the equation

$$dy/dt = k(1-y)^2, \tag{i}$$

where k is a positive constant. Show that $y = 1$ is the only critical point, with the corresponding equilibrium solution $\phi(t) = 1$.

(b) Sketch $f(y)$ versus y. Show that y is increasing as a function of t for $y < 1$ and also for $y > 1$. The phase line has upward-pointing arrows both below and above $y = 1$. Thus solutions below the equilibrium solution approach it, and those above it grow farther away. Therefore, $\phi(t) = 1$ is semistable.

(c) Solve Eq. (i) subject to the initial condition $y(0) = y_0$ and confirm the conclusions reached in part (b).

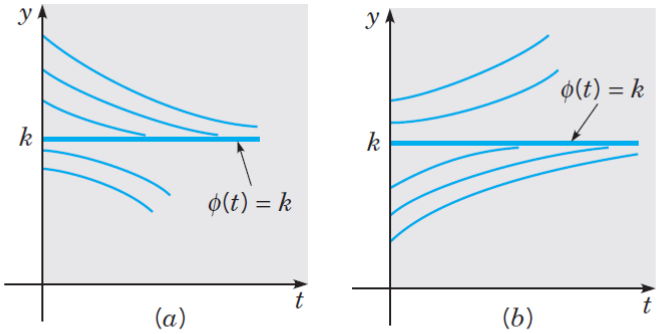

FIGURE 2.5.9 In both cases the equilibrium solution $\phi(t) = k$ is semistable. (a) $dy/dt \le 0$; (b) $dy/dt \ge 0$.