

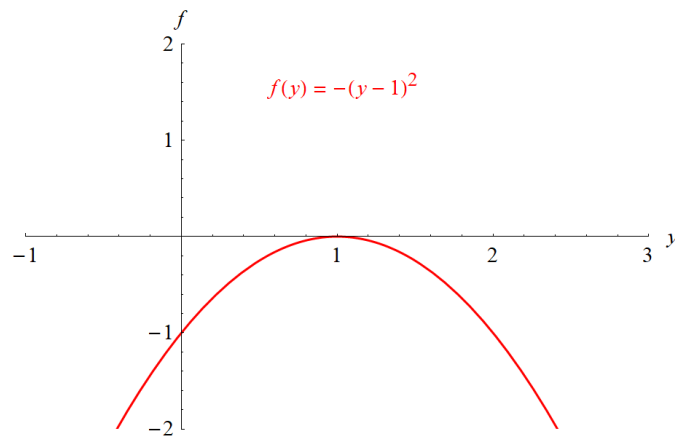
## Problem 8

Problems 8 through 13 involve equations of the form  $dy/dt = f(y)$ . In each problem sketch the graph of  $f(y)$  versus  $y$ , determine the critical (equilibrium) points, and classify each one asymptotically stable, unstable, or semistable (see Problem 7). Draw the phase line, and sketch several graphs of solutions in the  $ty$ -plane.

$$dy/dt = -k(y - 1)^2, \quad k > 0, \quad -\infty < y_0 < \infty$$

### Solution

In this problem  $f(y) = -k(y - 1)^2$ . Below is a graph of  $f(y)$  versus  $y$  for  $k = 1$ .

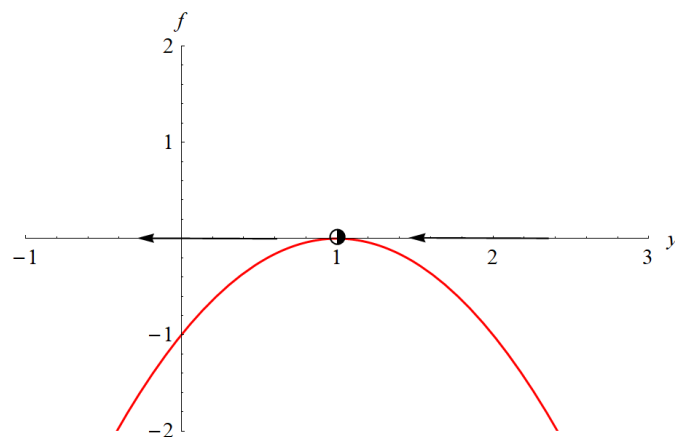


The equilibrium solutions are found by solving  $f(y) = 0$  for  $y$ .

$$-k(y - 1)^2 = 0$$

$$y = \{1\}$$

As indicated below by the half-filled circle,  $y = 1$  is stable from the right and unstable from the left. It is said to be semistable.



The arrows pointing left on the  $y$ -axis (phase line) mean that  $y$  is decreasing in time.

Some possible solution curves in the  $ty$ -plane for  $t \geq 0$  are shown below. At every point, they are tangent to the direction field vectors  $\langle 1, -(y-1)^2 \rangle$ .

