

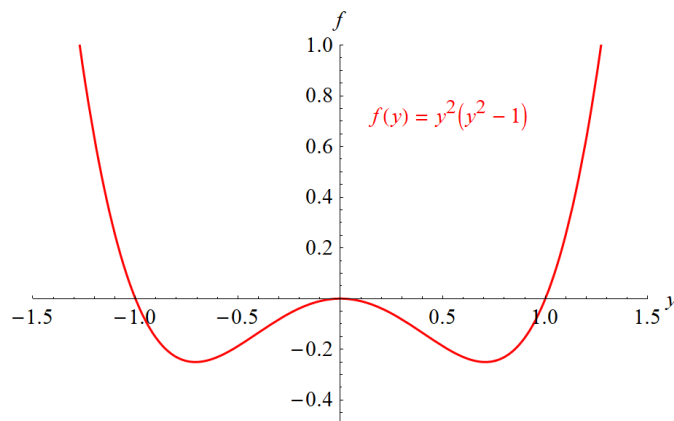
Problem 9

Problems 8 through 13 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus y , determine the critical (equilibrium) points, and classify each one asymptotically stable, unstable, or semistable (see Problem 7). Draw the phase line, and sketch several graphs of solutions in the ty -plane.

$$dy/dt = y^2(y^2 - 1), \quad -\infty < y_0 < \infty$$

Solution

In this problem $f(y) = y^2(y^2 - 1)$. Below is a graph of $f(y)$ versus y .



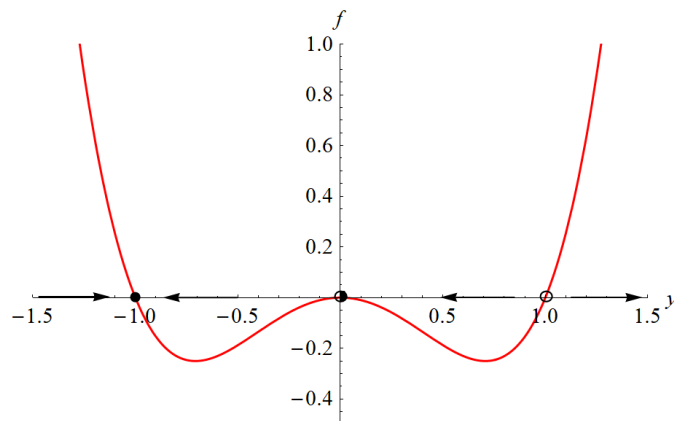
The equilibrium solutions are found by solving $f(y) = 0$ for y .

$$y^2(y^2 - 1) = 0$$

$$y^2 = 0 \quad \text{or} \quad y^2 - 1 = 0$$

$$y = \{-1, 0, 1\}$$

As indicated below by the half-filled circle, $y = 0$ is stable from the right and unstable from the left. It is said to be semistable. $y = -1$ is stable, and $y = 1$ is unstable.



The arrows pointing left and right on the y -axis (phase line) mean that y is decreasing and increasing in time, respectively.

Some possible solution curves in the ty -plane for $t \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, y^2(y^2 - 1) \rangle$.

