

Problem 2

In each of Problems 1 and 2, transform the given initial value problem into an equivalent problem with the initial point at the origin.

$$dy/dt = 1 - y^3, \quad y(-1) = 3$$

Solution

The aim is to make the initial condition $y(0) = 0$ by changing variables. Start by changing the independent variable t .

$$t' = t + 1$$

This changes the initial condition $y(t = -1) = 3$ to $y(t' = 0) = 3$ and the ODE to

$$dy/dt = 1 - y^3.$$

Use the chain rule to find what dy/dt is in terms of dy/dt' .

$$\frac{dy}{dt} = \frac{dy}{dt'} \frac{dt'}{dt} = \frac{dy}{dt'} (1) = \frac{dy}{dt'}$$

As a result of changing the independent variable, the initial value problem is

$$dy/dt' = 1 - y^3, \quad y(0) = 3.$$

Now change the dependent variable y .

$$y' = y - 3 \tag{1}$$

This changes the initial condition $y(t' = 0) = 3$ to $y'(t' = 0) = 0$ and the ODE to

$$dy'/dt' = 1 - (y' + 3)^3.$$

Differentiate both sides of equation (1) with respect to t' to find what dy'/dt' is in terms of dy'/dt' .

$$\frac{dy'}{dt'} = \frac{dy'}{dt'}$$

Therefore, the initial value problem can be written as

$$dy'/dt' = 1 - (y' + 3)^3, \quad y'(0) = 0.$$

Note that the primes here signify new variables and are not derivatives as usual.