

Problem 13

Let $\phi_n(x) = x^n$ for $0 \leq x \leq 1$ and show that

$$\lim_{n \rightarrow \infty} \phi_n(x) = \begin{cases} 0, & 0 \leq x < 1, \\ 1, & x = 1. \end{cases}$$

This example shows that a sequence of continuous functions may converge to a limit function that is discontinuous.