

Problem 4

In each of Problems 3 through 6, let $\phi_0(t) = 0$ and define $\{\phi_n(t)\}$ by the method of successive approximations

- (a) Determine $\phi_n(t)$ for an arbitrary value of n .
- (b) Plot $\phi_n(t)$ for $n = 1, \dots, 4$. Observe whether the iterates appear to be converging.
- (c) Express $\lim_{n \rightarrow \infty} \phi_n(t) = \phi(t)$ in terms of elementary functions; that is, solve the given initial value problem.
- (d) Plot $|\phi(t) - \phi_n(t)|$ for $n = 1, \dots, 4$. For each of $\phi_1(t), \dots, \phi_4(t)$, estimate the interval in which it is a reasonably good approximation to the actual solution.

$$y' = -y - 1, \quad y(0) = 0$$