

## Problem 5

In each of Problems 3 through 6, let  $\phi_0(t) = 0$  and define  $\{\phi_n(t)\}$  by the method of successive approximations

- (a) Determine  $\phi_n(t)$  for an arbitrary value of  $n$ .
- (b) Plot  $\phi_n(t)$  for  $n = 1, \dots, 4$ . Observe whether the iterates appear to be converging.
- (c) Express  $\lim_{n \rightarrow \infty} \phi_n(t) = \phi(t)$  in terms of elementary functions; that is, solve the given initial value problem.
- (d) Plot  $|\phi(t) - \phi_n(t)|$  for  $n = 1, \dots, 4$ . For each of  $\phi_1(t), \dots, \phi_4(t)$ , estimate the interval in which it is a reasonably good approximation to the actual solution.

$$y' = -y/2 + t, \quad y(0) = 0$$