Problem 1

In each of Problems 1 through 6, solve the given difference equation in terms of the initial value y_0. Describe the behavior of the solution as $n \to \infty$.

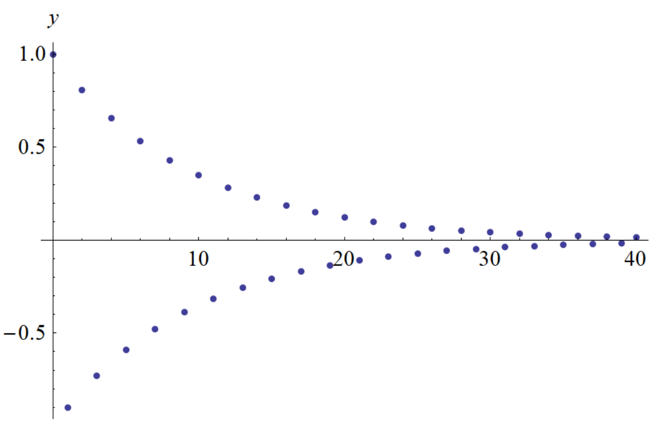
$$y_{n+1} = -0.9y_n$$

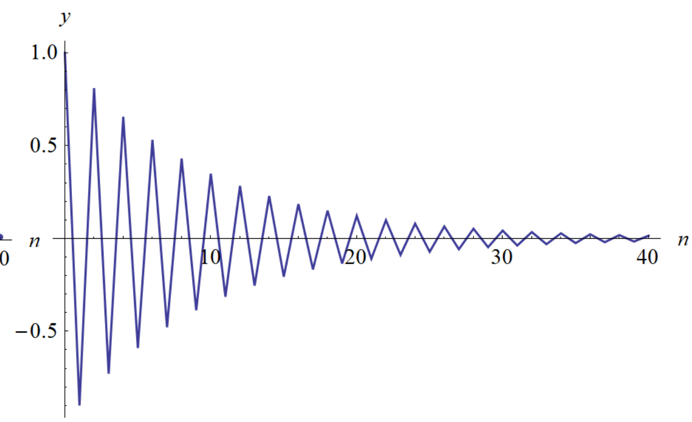
Solution

This is a first-order linear difference equation, so it can be solved by iteration.

$$\begin{aligned} n = 0: \quad & y_1 = -0.9y_0 \\ n = 1: \quad & y_2 = -0.9y_1 = -0.9(-0.9)y_0 \\ n = 2: \quad & y_3 = -0.9y_2 = -0.9(-0.9)(-0.9)y_0 \\ & \vdots \\ & y_n = (-0.9)^n y_0 \end{aligned}$$

Below on the left is a plot of y_n versus n for $y_0 = 1$. On the right is the same graph but with the dots connected in order to better illustrate the solution's behavior.





Now take the limit of y_n as $n \to \infty$.

$$\begin{aligned} \lim_{n\to\infty} y_n &= \lim_{n\to\infty} (-0.9)^n y_0 \\ &= y_0 \lim_{n\to\infty} (-1)^n (0.9)^n \\ &= y_0 \left[\lim_{n\to\infty} (-1)^n\right] \left[\lim_{n\to\infty} (0.9)^n\right] \\ &= y_0 \left[\lim_{n\to\infty} (-1)^n\right] \left[\lim_{n\to\infty} e^{\ln(0.9)^n}\right] \\ &= y_0 \left[\lim_{n\to\infty} (-1)^n\right] \left[\lim_{n\to\infty} e^{n\ln(0.9)}\right] \\ &= y_0 \left[\lim_{n\to\infty} (-1)^n\right] \left[\lim_{n\to\infty} e^{-n\ln(10/9)}\right] \\ &= y_0 \left[\lim_{n\to\infty} (-1)^n\right] (e^{-\infty}) \\ &= 0 \end{aligned}$$