Problem 4

In each of Problems 1 through 6, solve the given difference equation in terms of the initial value y_0. Describe the behavior of the solution as $n \to \infty$.

$$y_{n+1} = (-1)^{n+1} y_n$$

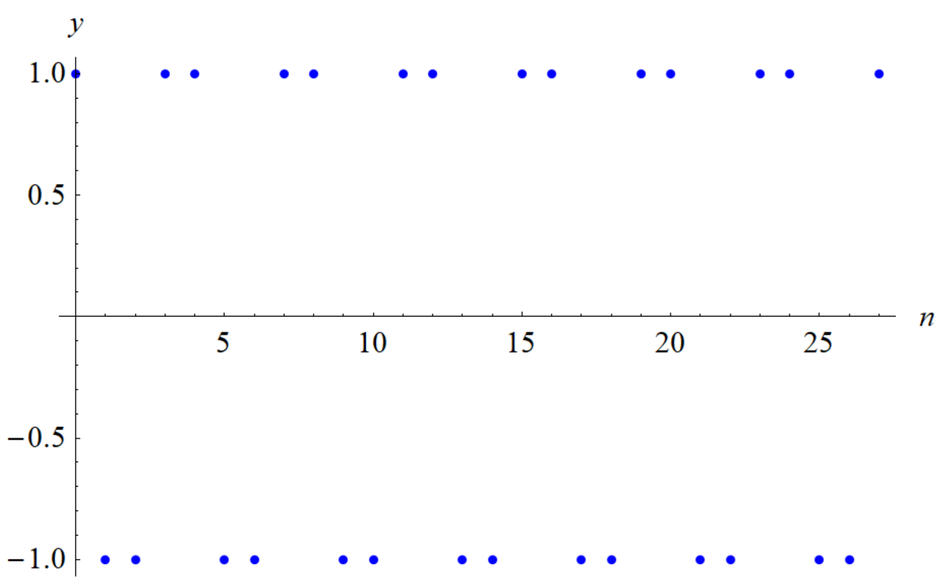
Solution

This is a first-order linear difference equation, so it can be solved by iteration.

$$\begin{aligned}
n = 0: &\quad y_1 = (-1)y_0 = -y_0 \\
n = 1: &\quad y_2 = (1)y_1 = (1)(-1)y_0 = -y_0 \\
n = 2: &\quad y_3 = (-1)y_2 = (-1)(1)(-1)y_0 = y_0 \\
n = 3: &\quad y_4 = (1)y_3 = (1)(-1)(1)(-1)y_0 = y_0 \\
&\ \ \vdots
\end{aligned}$$

$$y_n = \begin{cases} -y_0 & \text{if } n = 1, 5, 9, \ldots \\ -y_0 & \text{if } n = 2, 6, 10, \ldots \\ y_0 & \text{if } n = 3, 7, 11, \ldots \\ y_0 & \text{if } n = 0, 4, 8, \ldots \end{cases} = \begin{cases} -y_0 & \text{if } n = 4k+1 \\ -y_0 & \text{if } n = 4k+2 \\ y_0 & \text{if } n = 4k+3 \\ y_0 & \text{if } n = 4k \end{cases}, \quad k = 0, 1, 2, \ldots$$

Below is a plot of y_n versus n for $y_0 = 1$.



The limit of y_n as $n \to \infty$ is undefined because it does not tend to any particular value.