Problem 51

In each of Problems 48 through 51, solve the given initial value problem using the methods of Problems 36 through 47.

$$y'y'' - t = 0, \qquad y(1) = 2, \quad y'(1) = 1$$

Solution

Rewrite the left side of the ODE by using the chain rule.

$$\frac{1}{2}(2y')y'' - t = 0$$

$$\frac{1}{2}\frac{d}{dt}[(y')^2] - t = 0$$

Bring t to the right side and multiply both sides by 2.

$$\frac{d}{dt}[(y')^2] = 2t$$

Integrate both sides with respect to t.

$$(y')^2 = t^2 + C_1$$

Apply the second initial condition $y'(1) = 1$ here to determine C_1.

$$(1)^2 = 1^2 + C_1 \quad \rightarrow \quad C_1 = 0$$

So the previous equation becomes

$$(y')^2 = t^2.$$

Take the square root of both sides.

$$\frac{dy}{dt} = \pm t$$

For the second initial condition to remain satisfied, we choose the plus sign.

$$\frac{dy}{dt} = t$$

Integrate both sides with respect to t once more.

$$y(t) = \frac{1}{2}t^2 + C_2$$

Apply the first initial condition $y(1) = 2$ to determine C_2.

$$2 = \frac{1}{2}(1)^2 + C_2 \quad \rightarrow \quad C_2 = \frac{3}{2}$$

Therefore,

$$y(t) = \frac{1}{2}t^2 + \frac{3}{2}.$$