

Problem 13

In each of Problems 1 through 32, solve the given differential equation. If an initial condition is given, also find the solution that satisfies it.

$$\frac{dy}{dx} = 1 + 2x + y^2 + 2xy^2$$

Solution

Factor the right side.

$$\begin{aligned}\frac{dy}{dx} &= 1 + 2x + (1 + 2x)y^2 \\ &= (1 + 2x)(1 + y^2)\end{aligned}$$

Because the ODE is of the form $y' = f(x)g(y)$, it can be solved by separating variables.

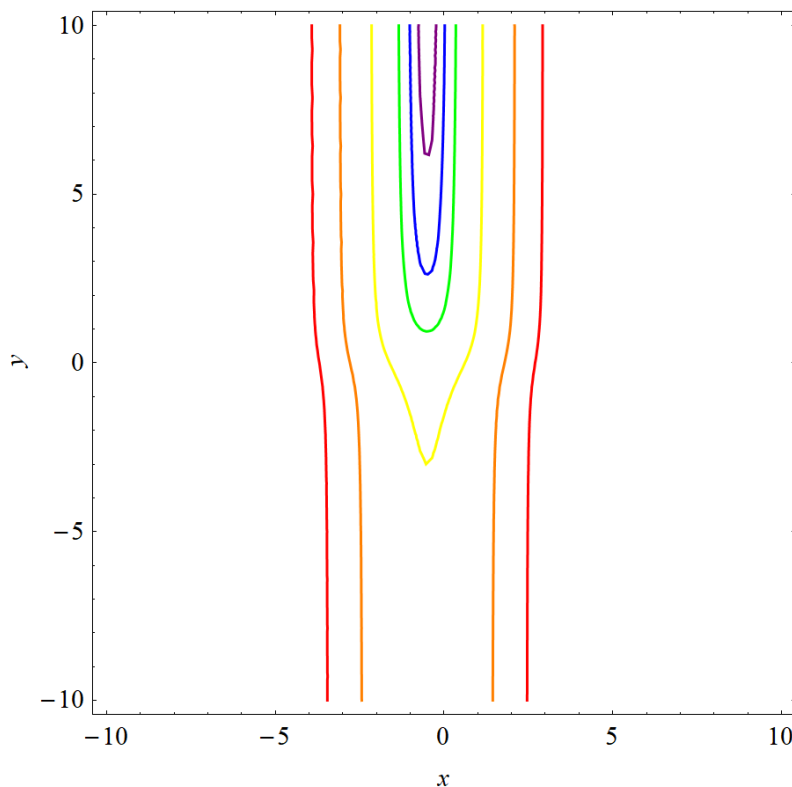
$$\frac{dy}{1 + y^2} = (1 + 2x) dx$$

Integrate both sides.

$$\tan^{-1} y = x + x^2 + C$$

Therefore,

$$\tan^{-1} y - x - x^2 = C$$



This figure illustrates several solutions of the family. In red, orange, yellow, green, blue, and purple are $C = -10$, $C = -5$, $C = -1$, $C = 1$, $C = 1.45$, and $C = 1.65$, respectively.