

## Problem 33

**Riccati Equations.** The equation

$$\frac{dy}{dt} = q_1(t) + q_2(t)y + q_3(t)y^2$$

is known as a Riccati<sup>23</sup> equation. Suppose that some particular solution  $y_1$  of this equation is known. A more general solution containing one arbitrary constant can be obtained through the substitution

$$y = y_1(t) + \frac{1}{v(t)}.$$

Show that  $v(t)$  satisfies the first order *linear* equation

$$\frac{dv}{dt} = -(q_2 + 2q_3y_1)v - q_3.$$

Note that  $v(t)$  will contain a single arbitrary constant.

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<sup>23</sup>Riccati equations are named for Jacopo Francesco Riccati (1676–1754), a Venetian nobleman, who declined university appointments in Italy, Austria, and Russia to pursue his mathematical studies privately at home. Riccati studied these equations extensively; however, it was Euler (in 1760) who discovered the result stated in this problem.