

Problem 35

The propagation of a single action in a large population (for example, drivers turning on headlights at sunset) often depends partly on external circumstances (gathering darkness) and partly on a tendency to imitate others who have already performed the action in question. In this case the proportion $y(t)$ of people who have performed the action can be described²⁴ by the equation

$$dy/dt = (1 - y)[x(t) + by], \quad (i)$$

where $x(t)$ measures the external stimulus and b is the imitation coefficient.

- (a) Observe that Eq. (i) is a Riccati equation and that $y_1(t) = 1$ is one solution. Use the transformation suggested in Problem 33, and find the linear equation satisfied by $v(t)$.
- (b) Find $v(t)$ in the case that $x(t) = at$, where a is a constant. Leave your answer in the form of an integral.

²⁴See Anatol Rapoport, "Contribution to the Mathematical Theory of Mass Behavior: I. The Propagation of Single Acts," *Bulletin of Mathematical Biophysics* 14 (1952), pp. 159-169, and John Z. Hearon, "Note on the Theory of Mass Behavior," *Bulletin of Mathematical Biophysics* 17 (1955), pp. 7-13.