Problem 3

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$6y'' - y' - y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$6(r^2 e^{rt}) - re^{rt} - e^{rt} = 0$$

Divide both sides by e^{rt}.

$$6r^2 - r - 1 = 0$$

$$(3r + 1)(2r - 1) = 0$$

$$r = \left\{-\frac{1}{3}, \frac{1}{2}\right\}$$

Two solutions to the ODE are $y = e^{-t/3}$ and $y = e^{t/2}$. Therefore, the general solution is

$$y(t) = C_1 e^{-t/3} + C_2 e^{t/2},$$

a linear combination of the two.