Problem 9

In each of Problems 9 through 16, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$y'' + y' - 2y = 0, \qquad y(0) = 1, \quad y'(0) = 1$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} + re^{rt} - 2(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + r - 2 = 0$$

$$(r + 2)(r - 1) = 0$$

$$r = \{-2, 1\}$$

Two solutions to the ODE are $y = e^{-2t}$ and $y = e^t$, so the general solution is

$$y(t) = C_1 e^{-2t} + C_2 e^t,$$

a linear combination of the two. Differentiate it once with respect to t.

$$y'(t) = -2C_1 e^{-2t} + C_2 e^t$$

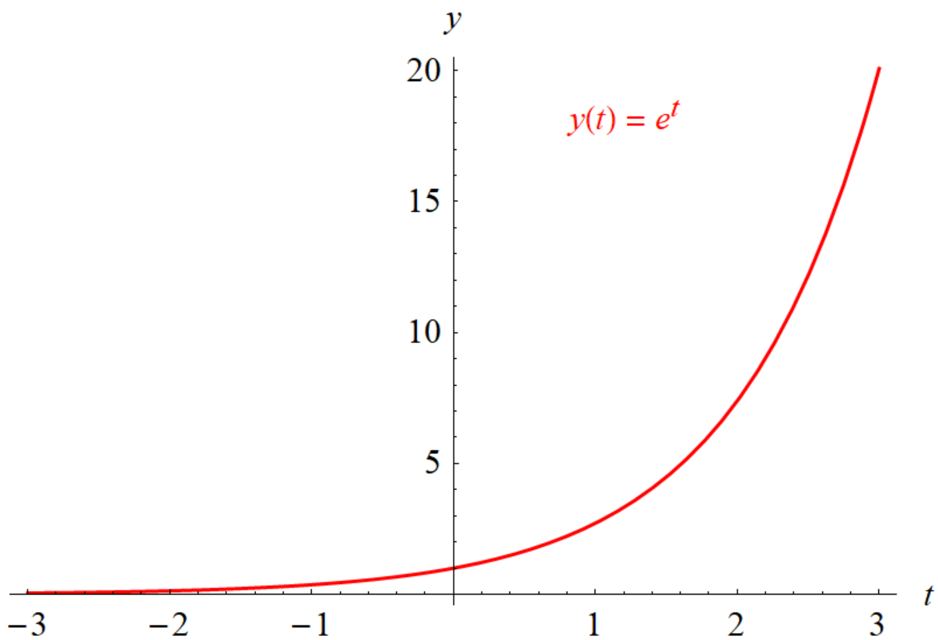
Apply the two initial conditions now to determine C_1 and C_2.

$$y(0) = C_1 + C_2 = 1$$
$$y'(0) = -2C_1 + C_2 = 1$$

Solving the system of equations yields $C_1 = 0$ and $C_2 = 1$. Therefore,

$$y(t) = e^t.$$

This solution diverges to ∞ as $t \to \infty$.



y
20
15
10
5
$y(t) = e^t$
−3
−2
−1
1
2
3
t