Problem 15

In each of Problems 9 through 16, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$y'' + 8y' - 9y = 0, \qquad y(1) = 1, \quad y'(1) = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \to \quad y' = re^{rt} \quad \to \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} + 8(re^{rt}) - 9(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + 8r - 9 = 0$$

$$(r + 9)(r - 1) = 0$$

$$r = \{-9, 1\}$$

Two solutions to the ODE are $y = e^{-9t}$ and $y = e^t$, so the general solution is

$$y(t) = C_1 e^{-9t} + C_2 e^t,$$

a linear combination of the two. Differentiate it once with respect to t.

$$y'(t) = -9C_1 e^{-9t} + C_2 e^t,$$

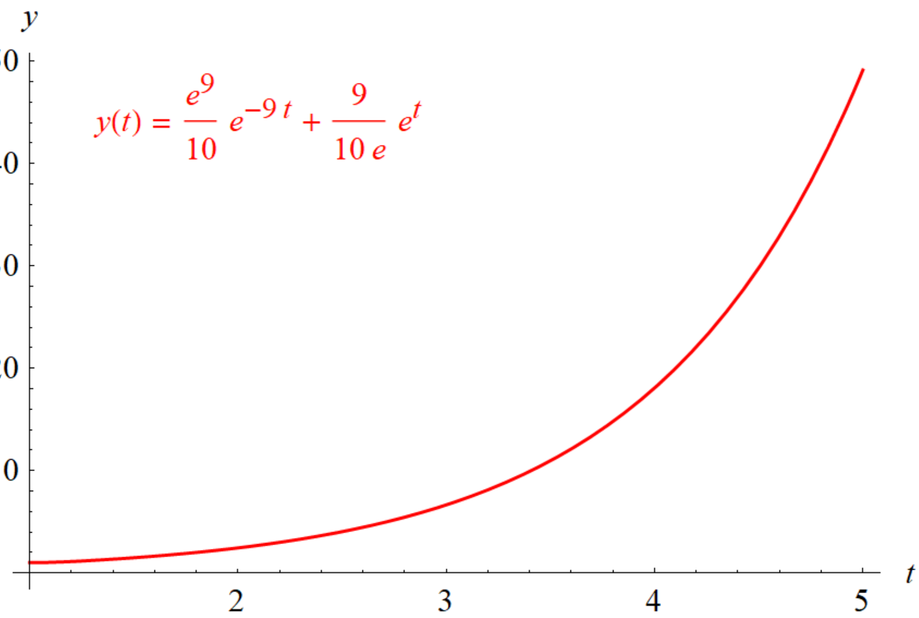
Apply the two initial conditions now to determine C_1 and C_2.

$$y(1) = C_1 e^{-9} + C_2 e^1 = 1$$
$$y'(1) = -9C_1 e^{-9} + C_2 e^1 = 0$$

Solving the system of equations yields $C_1 = e^9/10$ and $C_2 = 9/(10e)$. Therefore,

$$y(t) = \frac{e^9}{10} e^{-9t} + \frac{9}{10e} e^t.$$

Because the coefficient of t in the second exponential function is positive, this solution diverges to ∞ as $t \to \infty$.


y
50
40
30
20
10
$y(t) = \frac{e^9}{10} e^{-9t} + \frac{9}{10e} e^t$
2
3
4
5
t