Problem 19

Find the solution of the initial value problem

$$y'' - y = 0, \qquad y(0) = \frac{5}{4}, \quad y'(0) = -\frac{3}{4}.$$

Plot the solution for $0 \leq t \leq 2$ and determine its minimum value.

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} - e^{rt} = 0$$

Divide both sides by e^{rt}.

$$r^2 - 1 = 0$$

$$r = \{-1, 1\}$$

Two solutions to the ODE are $y = e^{-t}$ and $y = e^t$, so the general solution is

$$y(t) = C_1 e^{-t} + C_2 e^t,$$

a linear combination of the two. Differentiate it once with respect to t.

$$y'(t) = -C_1 e^{-t} + C_2 e^t$$

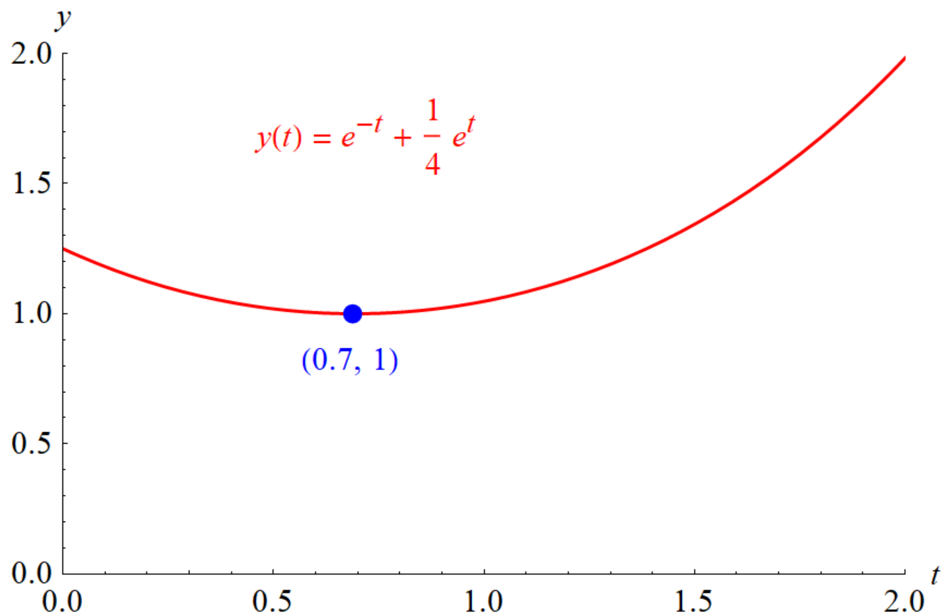
Apply the two initial conditions now to determine C_1 and C_2.

$$y(0) = C_1 + C_2 = \frac{5}{4}$$

$$y'(0) = -C_1 + C_2 = -\frac{3}{4}$$

Solving the system of equations yields $C_1 = 1$ and $C_2 = 1/4$. Therefore,

$$y(t) = e^{-t} + \frac{1}{4} e^t.$$


y
2.0
1.5
1.0
0.5
0.0
$y(t) = e^{-t} + \frac{1}{4} e^{t}$
(0.7, 1)
0.0
0.5
1.0
1.5
2.0
t